

Influence of the second viscosity on the flow propagation

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Abstract

We found the energetic stability criteria of non-equilibrium gas plain flows.

The second (volume) viscosity coefficient ζ can be negative in nonequilibrium media. Medium with the negative second viscosity is acoustically unstable. The nonlinear structures in the acoustically active gases is strongly different from that in the equilibrium media. In present paper we investigate some problems, connected with the second viscosity influence on flow characteristics.

Let us consider the thin body ($l_x/l_y \gg M$), $M > 1$ is the Mach number in the supersonic laminated stream of nonequilibrium gas. It is proposed that the angle of attack $\delta = l_y/l_x \ll 1$. It follows in the standard way ([Landau et al. 1988]) that a system of relaxation gasdynamics equations reduces to the equation

$$\frac{\partial u}{\partial y} \pm \beta \frac{\partial u}{\partial x} \pm \frac{\gamma M^3}{2\beta u_s} u \frac{\partial u}{\partial x} = \pm \frac{\mu M^3}{u_s} \frac{\partial^2 u}{\partial x^2} \mp \frac{M\alpha}{2\beta} u$$

with the boundary conditions

$$u(y = \pm 0) = \pm \frac{Mu_s}{\beta} \left[\frac{\partial \xi_{2,1}}{\partial x} + \frac{M^3}{2\beta^2} \frac{\partial^2 \xi_{2,1}}{\partial x^2} + \frac{\alpha M}{2\beta^2} \xi_{2,1} \right]. \quad (1)$$

Here $u = \partial \phi / \partial x$, ϕ is velocity disturbances potential, $\beta = \sqrt{M^2 - 1}$, $\alpha = \alpha(\zeta) < 0$ is an acoustic increment, $y = \xi_{2,1}(x)$ are equations of upper and lower body surface.

Using equation (1) we obtain the coefficients of resistance C_x and lifting force C_y . In the limit case of homogeneous nonequilibrium particles distribution the expressions for these coefficients is reduced to form:

$$C_x = \frac{2}{\beta} \left[2\delta^2 + \langle \theta_1^2 \rangle + \langle \theta_2^2 \rangle + \frac{M\delta^2}{2\beta} \alpha l_x \right],$$

$$C_y = \frac{2}{\beta} \left[2\delta + \frac{M\delta}{\beta^2} \alpha l_x \right],$$

where $\theta_{2;1}(x) = \partial \xi_{2;1}(x)/\partial x - \delta$, $\xi_{2;1}(0) = l_y$, $\xi_{2;1}(l_x) = 0$.

Our main result is some decrease of C_x and C_y at $\alpha < 0$, i.e. in media with negative second viscosity.

The result, presented above, is only correct for laminated streams. In the nonequilibrium media the bounds of laminar region is changed. The traditional neglect of the second viscosity in subsonics is not correct if $\zeta \gg \eta$, where η , is the shear viscosity coefficient. For example, in CO_2, N_2O - gases the relation $\zeta/\eta > 10^3$, as $T > 300K$. ([Emanuel 1990]). Therefore, the second viscosity is essential for the sound propagation and the supersonic flows.

It is shown that such neglect is only correct under following conditions

$$|\zeta|/\eta \ll 1/M^2, R \geq 1; |\zeta|/\eta \ll R/M^2, R \ll 1.$$

where R is Reynolds number. In other cases the second viscosity can even be impotent for subsonic flows.

Taking into account these conditions the critical Reynolds number is obtained

$$R_c = \min|(D + B)/T|, \quad (2)$$

where

$$D = \int v_{ik}^2 dV, B = \frac{\zeta}{\eta} \int v_{ii}^2 dV, T = \int v_i v_k v_{ik} dV,$$

$v_{ik} = \partial v_i / \partial x_k$, \vec{v} is a disturbance of the main flow.

At $\zeta = 0$, condition (2) is similar to one from the ([Landau et al. 1988]). At $\zeta > 0$, the instability threshold increases comparatively with the ([Landau et al. 1988]). Otherwise at $\zeta < 0$, but $|B| < D$ the instability threshold decreases. Finally, at $|B| > D$ the critical number does not exist. Moreover, at $R < R_c$ the flow is unstable relatively to every low frequency disturbances.

The condition $|B| \geq D$ is satisfied with Mach numbers $M \geq (\eta/|\zeta|)^{1/4}$ if $R \geq 1$ or $M \geq (R^2 \eta/|\zeta|)^{1/4}$ if $R \ll 1$.

References

- [Landau et al. 1988] Landau LD, Lifshitz EM (1988) Hydrodynamics. Nauka 137–143
- [Emanuel 1990] Emanuel G (1990) Phys. Fluids A 2: 2252–2254